Figure 6.1 Closed-loop poles and response: a. stable system; b. unstable system



Common cause of problems in finding closed-loop poles:

a. original system;**b.** equivalent

system



Figure 6.3 Equivalent closedloop transfer function



<i>s</i> ⁴	a_4	a_2	a_0
s ³	<i>a</i> ₃	a_1	0
s^2			
s^1			
s^0			

Table 6.1Initial layout for Routh table

<i>s</i> ⁴	a_4	<i>a</i> ₂	a_0
<i>s</i> ³	<i>a</i> ₃	a_1	0
s ²	$\frac{-\begin{vmatrix} a_4 & a_2 \\ a_3 & a_1 \end{vmatrix}}{a_3} = b_1$	$\frac{-\begin{vmatrix} a_4 & a_0 \\ a_3 & 0 \end{vmatrix}}{a_3} = b_2$	$\frac{-\begin{vmatrix} a_4 & 0 \\ a_3 & 0 \end{vmatrix}}{a_3} = 0$
s^1	$\frac{-\begin{vmatrix} a_3 & a_1 \\ b_1 & b_2 \end{vmatrix}}{b_1} = c_1$	$\frac{-\begin{vmatrix} a_3 & 0 \\ b_1 & 0 \end{vmatrix}}{b_1} = 0$	$\frac{-\begin{vmatrix} a_3 & 0 \\ b_1 & 0 \end{vmatrix}}{b_1} = 0$
s^0	$\frac{-\begin{vmatrix} b_1 & b_2 \\ c_1 & 0 \end{vmatrix}}{c_1} = d_1$	$\frac{-\begin{vmatrix} b_1 & 0 \\ c_1 & 0 \end{vmatrix}}{c_1} = 0$	$\frac{-\begin{vmatrix} b_1 & 0 \\ c_1 & 0 \end{vmatrix}}{c_1} = 0$

Table 6.2Completed Routh table

Figure 6.4 a. Feedback system for Example 6.1; b. equivalent closed-loop system



Table 6.3Completed Routh table for Example 6.1

<i>s</i> ³	1	31	0
<i>s</i> ²	-10-1	1030 103	0
s^1	$\frac{-\begin{vmatrix} 1 & 31 \\ 1 & 103 \end{vmatrix}}{1} = -72$	$\frac{-\begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix}}{1} = 0$	$\frac{-\begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix}}{1} = 0$
s^0	$\frac{-\begin{vmatrix} 1 & 103 \\ -72 & 0 \end{vmatrix}}{-72} = 103$	$\frac{-\begin{vmatrix} 1 & 0 \\ -72 & 0 \end{vmatrix}}{-72} = 0$	$\frac{-\begin{vmatrix} 1 & 0 \\ -72 & 0 \end{vmatrix}}{-72} = 0$

s ⁵	1	3	5
<i>s</i> ⁴	2	6	3
s ³	Χ ε	$\frac{7}{2}$	0
<i>s</i> ²	$\frac{6\epsilon-7}{\epsilon}$	3	0
s^1	$\frac{42\epsilon - 49 - 6\epsilon^2}{12\epsilon - 14}$	0	0
<i>s</i> ⁰	3	0	0

Table 6.4Completed Routh table for Example 6.2

Label	First Column	$\epsilon = +$	e = -
s ⁵	1	+	+
s ⁴	2	+	+
s ³	Χ ε	+	_
s ²	$\frac{6\epsilon-7}{\epsilon}$	_	+
s^1	$\frac{42\epsilon - 49 - 6\epsilon^2}{12\epsilon - 14}$	+	+
<i>s</i> ⁰	3	+	+

Table 6.5

Determining signs in first column of a Routh table with zero as first element in a row

s ⁵	3	6	2
<i>s</i> ⁴	5	3	1
s^3	4.2	1.4	
s^2	1.33	1	
s^1	-1.75		
s^0	1		

Table 6.6Routh table for Example 6.3

Table 6.7Routh table for Example 6.4

s ⁵	1	6	8
s ⁴	X 1	42 6	56 8
<i>s</i> ³	X X 1	X 123	X X 0
s^2	3	8	0
s^1	$\frac{1}{3}$	0	0
<i>s</i> ⁰	8	0	0

Figure 6.5 Root positions to generate even polynomials: *A* , *B*, *C*, or any combination



<i>s</i> ⁸	1	12	39	48	20
<i>s</i> ⁷	1	22	59	38	0
s ⁶	-10 - 1	-20-2	1 0 1	202	0
s ⁵	201	603	40 2	0	0
<i>s</i> ⁴	1	3	2	0	0
<i>s</i> ³	X X 2	X K 3	かん0	0	0
<i>s</i> ²	$\frac{3}{2}$ 3	X 4	0	0	0
s^1	$\frac{1}{3}$	0	0	0	0
s^0	4	0	0	0	0

Table 6.8Routh table for Example 6.5

Table 6.9 Summary of pole locations for Example 6.5 Polynomial

Location	Even (fourth-order)	Other (fourth-order)	Total (eighth-order)
Right half-plane	0	2	2
Left half-plane	0	2	2
jω	4	0	4

Table 6.9

Summary of pole locations for Example 6.5

Figure 6.6 Feedback control system for Example 6.6



Table 6.10Routh table for Example 6.6

s^4	1	11	200
s ³	K 1	K 1	
s^2	1 10 1	200 20	
s^1	-19		
s^0	20		

Figure 6.7 Feedback control system for Example 6.7



s ⁵	2	2	2
<i>s</i> ⁴	3	3	1
s ³	ЮE	$\frac{4}{3}$	
s ²	$\frac{3\epsilon-4}{\epsilon}$	1	
s^1	$\frac{12\epsilon - 16 - 3\epsilon^2}{9\epsilon - 12}$		
<i>s</i> ⁰	1		

 Table 6.11
 Routh table for Example 6.7

s ⁵	1	3	3
s^4	2	2	2
s ³	2	2	
<i>s</i> ²	<i>X</i> ε	2	
s^1	$\frac{2\epsilon-4}{\epsilon}$		
s^0	2		

Table 6.12Alternative Routh table for Example 6.7

Figure 6.8 Feedback control system for Example 6.8



Table 6.13Routh table for Example 6.8

<i>s</i> ⁸	1	10	48	128	128
<i>s</i> ⁷	X 1	24 8	96 32	192 64	
s ⁶	21	JC 8	.64 32	128 64	
s ⁵	8-63	A 32 16	A 64 32	A A 0	
<i>s</i> ⁴	$\frac{8}{3}$ 1	$\frac{64}{3}$ 8	.64 24		
<i>s</i> ³	-8 -1	-40 -5			
s^2	X 1	.24 8			
s^1	3				
<i>s</i> ⁰	8				

Table 6.14Summary of pole locations for Example 6.8

Polynomial

Location	Even	Other	Total
	(sixth-order)	(second-order)	(eighth-order)
Right half-plane	2	0	2
Left half-plane	2	2	4
$j\omega$	2	0	2

Note: rhp = right half-plane, lhp = left half-plane.

Table 6.14

Summary of pole locations for Example 6.5

Jason is an underwater, remotecontrolled vehicle that has been used to explore the wreckage of the *Lusitania*. The manipulator and camera comprise some of the vehicle's control systems.



Courtesy of Woods Hole Oceanographic Institution.

Figure 6.10 Feedback control system for Example 6.9



Figure 6.11 The FANUC Robot M- 400 can be configured for 4- or 5-axis of motion. It is seen here moving and stacking boxes.



Courtesy of Fanuc Robotics.

<i>s</i> ³	1	77
s^2	18	K
s^1	$\frac{1386 - K}{18}$	
s^0	K	

Table 6.15Routh table for Example 6.9

<i>s</i> ³	1	77
s^2	18	1386
s^1	X 36	
s^0	1386	

Table 6.16Routh table forExample 6.9 with K = 1386

Table 6.17Routh table for Example 6.10

<i>s</i> ⁴	1	30	200
s ³	X 1	.30 10	
s^2	201	200 10	
s^1	1 € 2	₽ 0	
<i>s</i> ⁰	10		

Table 6.18Routh table for Example 6.11



Table 6.19Routh table for antenna controlcase study

<i>s</i> ³	1	171
s^2	101.71	6.63 <i>K</i>
s^1	17392.41 – 6.63 <i>K</i>	0
s ⁰	6.63 <i>K</i>	

Table 6.20Routh table for UFSS case study

<i>s</i> ⁴	1	3.457	$0.0416 + 0.109K_1$
<i>s</i> ³	3.456	$0.719 + 0.25K_1$	
s^2	$11.228 - 0.25K_1$	$0.144 + 0.377K_1$	
s^1	$\frac{-0.0625K_1^2 + 1.324K_1 + 7.575}{11.228 - 0.25K_1}$		
<i>s</i> ⁰	$0.144 + 0.377K_1$		

Note: Some rows have been multiplied by a positive constant for convenience.



$$\frac{R(s)}{s^4 + 4s^3 + 8s^2 + 20s + 15} \xrightarrow{C(s)}$$

















Figure P6.10 Closed-loop system with pole plot



Aircraft pitch loop model



Block diagram of a chemical process-control system





Courtesy of Kazuhiko Kawamura, Vanderbilt University.

(*a*)



Figure P6.13 a. Soft Arm used for feeding; b. simplified block

diagram

Figure P6.14 Towed vehicle roll

control



Figure P6.15 Cutting force control system



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a. A magnetic levitation transportation system; **b.** simplified block diagram (©1998 IEEE)

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(*a*)

